Water Resour Manage (2007) 21:801–819
DOI 10.1007/s11269-006-9062-y

ORIGINAL ARTICLE

Drought forecasting using the Standardized Precipitation Index

A. Cancelliere · G. Di Mauro · B. Bonaccorso · G. Rossi

Received: 24 January 2006 / Accepted: 7 June 2006 / Published online: 12 December 2006

Abstract Unlike other natural disasters, drought events evolve slowly in time and their impacts generally span a long period of time. Such features do make possible a more effective drought mitigation of the most adverse effects, provided a timely monitoring of an incoming drought is available.

Among the several proposed drought monitoring indices, the Standardized Precipitation Index (SPI) has found widespread application for describing and comparing droughts among different time periods and regions with different climatic conditions. However, limited efforts have been made to analyze the role of the SPI for drought forecasting.

The aim of the paper is to provide two methodologies for the seasonal forecasting of SPI, under the hypothesis of uncorrelated and normally distributed monthly precipitation aggregated at various time scales k. In the first methodology, the auto-covariance matrix of SPI values is analytically derived, as a function of the statistics of the underlying monthly precipitation process, in order to compute the transition probabilities from a current drought condition to another in the future. The proposed analytical approach appears particularly valuable from a practical stand point in light of the difficulties of applying a frequency approach due to the limited number of transitions generally observed even on relatively long SPI records. Also, an analysis of the applicability of a Markov chain model has revealed the inadequacy of such an approach, since it leads to significant errors in the transition probability as shown in the paper. In the second methodology, SPI forecasts at a generic time horizon M are analytically determined, in terms of conditional expectation, as a function of past values of monthly precipitation. Forecasting accuracy is estimated through an expression of the Mean Square Error, which allows one to derive confidence intervals of prediction. Validation of the derived expressions is carried out by comparing theoretical forecasts and observed SPI values by means of a moving window technique. Results seem to confirm the reliability of the proposed methodologies, which therefore can find useful application within a drought monitoring system.

A. Cancelliere · G. Di Mauro · B. Bonaccorso (✉) · G. Rossi
Department of Civil and Environmental Engineering, University of Catania, V.le A. Doria, 6-95125 Catania, Italy
e-mail: bbonacco@dica.unict.it

Keywords Drought · Precipitation · SPI · Stochastic techniques · Transition probabilities · Forecast

Introduction

Due to a slow evolution in time, drought is a phenomenon whose consequences take a significant amount of time with respect to its inception in order to be perceived by the socio-economic systems. Taking advantage of this feature, an effective mitigation of the most adverse drought impacts is possible, more than in the case of other extreme hydrological events such as floods, earthquakes, hurricanes, etc., provided a drought monitoring system, able to promptly warn of the onset of a drought and to follow its evolution in space and time, is in operation (Rossi, 2003). To this end, an accurate selection of indices for drought identification, providing a synthetic and objective description of drought conditions, represents a key point for the implementation of an efficient drought watch system.

Among the several proposed indices for drought monitoring, the Standardized Precipitation Index (SPI) has found widespread application (McKee *et al*., 1993; Heim, 2000; Wilhite *et al*., 2000; Rossi and Cancelliere, 2002). Guttman (1998) and Hayes *et al*. (1999) compared SPI with Palmer Drought Severity Index (PDSI) and concluded that the SPI has advantages of statistical consistency, and the ability to describe both short-term and long-term drought impacts through the different time scales of precipitation anomalies. Also, due to its intrinsic probabilistic nature, the SPI is the ideal candidate for carrying out drought risk analysis (Guttmann, 1999). An evaluation of common indicators, according to six weighted evaluation criteria of performance (robustness, tractability, transparency, sophistication, extendability, and dimensionality), indicates strengths of the SPI and Deciles over the PDSI (Keyantash and Dracup, 2002).

Although most of the indices have been developed with the intent to monitor current drought conditions, nevertheless some of them can be used to forecast the possible evolution of an ongoing drought, in order to adopt appropriate mitigation measures and policies for water resources management. Within this framework, Karl *et al*. (1986) assessed the amount of precipitation needed to restore normal conditions after a drought event, with reference to the Palmer Hydrologic Drought Index (PHDI). Cancelliere *et al*. (1996) proposed a procedure for short-middle term forecasting of the Palmer Index and tested its applicability to Mediterranean regions, by computing the probability that an ongoing drought will end in the following months. Other authors (Lohani *et al*., 1998) proposed a forecasting procedure of the Palmer index based on the first-order Markov chains, which enables one to forecast drought conditions for future months, based on the current drought class described by the PHDI values. Recently, Bordi *et al*. (2005) compared two stochastic techniques, namely an autoregressive model and a novel method called Gamma Highest Probability (GAHP), for forecasting SPI series at lag 1. The latter method forecasts precipitation of the next month as the mode of a Gamma distribution fitted to the observed precipitation series. They concluded that the GAHP performs better, especially in spring and summer months.

In the present paper, a seasonal forecast of the SPI is addressed by means of stochastic techniques. In particular, transition probabilities from a drought class to another at different time horizons are analytically derived as a function of the statistical properties of the underlying monthly precipitation. The usefulness of such analytical derivation for estimating transition probabilities is evident in light of the fact that a Markov chain approach is not adequate to model SPI series, as it is demonstrated in a following section. Also, the analytical approach enables one to overcome the difficulties related to a frequency approach, whose

Table 1 Wet and drought period classification according to the SPI index

	Index value	Class	Probability	ΔP
Non Drought	$\text{SPI} \geq 2.00$	Extremely wet	0.977–1.000	0.023
	$1.50 \leq \text{SPI} < 2.00$	Very wet	0.933–0.977	0.044
	$1.00 \leq \text{SPI} < 1.50$	Moderately wet	0.841–0.933	0.092
	$-1.00 \leq \text{SPI} < 1.00$	Near normal	0.159–0.841	0.682
	$-1.50 \leq \text{SPI} < -1.00$	Moderate drought	0.067–0.159	0.092
	$-2.00 \leq \text{SPI} < -1.50$	Severe drought	0.023–0.067	0.044
	$\text{SPI} < -2.00$	Extreme drought	0.000–0.023	0.023

reliability may be hindered by the generally limited sample size of the available precipitation series.

The spatial variability of transition probabilities in Sicily is analyzed in order to provide indications about the different behaviour of drought phenomena in different areas of the island.

Also a model to evaluate SPI forecast on the basis of past values of precipitation has been developed. More specifically, analytical expressions of short-middle term forecasts of the SPI are derived as the expectation of future SPI values conditioned on past monthly precipitation, under the hypothesis of uncorrelated and normally distributed precipitation aggregated at different time scales k. The accuracy of the model is evaluated in terms of the Mean Square Error (MSE) of prediction (Brockwell and Davis, 1996), which allows confidence intervals for forecasted values to be computed. Forecasting future values in terms of conditional expectation ensures that the corresponding forecasts will have minimum MSE. Validation of the model is carried out based on the historical series observed at 43 precipitation stations in Sicily (Italy), making use of a moving window scheme for parameters estimation.

The standardized precipitation index

The SPI is able to take into account the different time scales at which the drought phenomenon occurs and, because of its standardization, is particularly suited to compare drought conditions among different time periods and regions with different climatic conditions (Bonaccorso *et al.*, 2003).

The index is based on an equi-probability transformation of aggregated monthly precipitation into a standard normal variable. In practice, computation of the index requires fitting a probability distribution to aggregated monthly precipitation series (e.g. $k = 3, 6, 12, 24$ months, etc.), computing the non-exceedence probability related to such aggregated values and defining the corresponding standard normal quantile as the SPI. McKee *et al.* (1993) assumed an aggregated precipitation gamma distributed and used a maximum likelihood method to estimate the parameters of the distribution.

Although McKee *et al.* (1993) originally proposed a classification restricted only to drought periods, it has become customary to use the index to classify wet periods as well. Table 1 reports the climatic classification according to the SPI, provided by the National Drought Mitigation Center (NDMC, http://drought.unl.edu). Also, the probabilities ΔP, that the index lies within each class are listed. Since our present work focuses on forecasting drought conditions, the near normal and wet classes have been grouped into one class termed "Non-drought".

Analytical derivation of transition probabilities of drought classes

Let $Z_{\nu,\tau}^{(k)}$ indicate the SPI value at year ν and month $\tau = 1, 2, \ldots 12$, for an aggregation time scale k of monthly precipitation. Also, let's indicate by C_i the generic drought class, for instance C_1 = Extreme, C_2 = Severe, C_3 = Moderate, C_4 = Non-drought. The probability that the SPI value after M months lies within a class C_j given that the SPI value at the current month lies within a class C_i, can be expressed as (Mood *et al.*, 1974):

$$P\left[Z_{\nu,\tau+M}^{(k)} \in C_j \,\middle|\, Z_{\nu,\tau}^{(k)} \in C_i\right] = \frac{\iint_{C_i,C_j} f_{Z_{\nu,\tau}^{(k)},Z_{\nu,\tau+M}^{(k)}}(t,s)\cdot dt\cdot ds}{\int_{C_i} f_{Z_{\nu,\tau}^{(k)}}(t)\cdot dt} \qquad (1)$$

where $f_{Z_{\nu,\tau}^{(k)},Z_{\nu,\tau+M}^{(k)}}(\cdot)$ is the joint density function of $Z_{\nu,\tau}^{(k)}$ and $Z_{\nu,\tau+M}^{(k)}$, $f_{Z_{\nu,\tau}^{(k)}}(\cdot)$ is the marginal density function of $Z_{\nu,\tau}^{(k)}$, t and s are integration dummy variables, and the integrals are extended to the range of each drought class.

Since, by definition, SPI is marginally distributed as a standard normal variable, it is fair to assume the joint density function in Equation (1) to be bivariate normal, namely :

$$f_{Z_{\nu,\tau}^{(k)},Z_{\nu,\tau+M}^{(k)}}(t,s) = \frac{1}{2\pi|\Sigma|}\cdot\exp\left(-\frac{1}{2}\mathrm{X}^T\Sigma^{-1}\mathrm{X}\right) \qquad (2)$$

where $\mathbf{X} = [t, s]^T$, and Σ represents the variance-covariance matrix:

$$\Sigma = \begin{bmatrix} 1 & \mathrm{cov}\left[Z_{\nu,\tau}^{(k)}, Z_{\nu,\tau+M}^{(k)}\right] \\ \mathrm{cov}\left[Z_{\nu,\tau}^{(k)}, Z_{\nu,\tau+M}^{(k)}\right] & 1 \end{bmatrix} \qquad (3)$$

Thus, the computation of transition probabilities in Equation (1) requires the determination of the autocovariance at lag M of $Z_{\nu,\tau+M}^{(k)}$ namely cov $[Z_{\nu,\tau}^{(k)}, Z_{\nu,\tau+M}^{(k)}]$.

Although in principle, such autocovariance could be estimated from an available sample, it is of interest here to derive its expression as a function of the statistics of the underlying precipitation. In general terms, such derivation is not straightforward, because of the equi-probability transformation underlying the SPI computation. However, under the hypothesis of monthly precipitation aggregated at time scale k normally distributed, the corresponding value of SPI can be computed through a simple standardization procedure:

$$Z_{\nu,\tau}^{(k)} = \frac{Y_{\nu,\tau}^{(k)} - \mu_\tau^{(k)}}{\sigma_\tau^{(k)}} \qquad (4)$$

with $Y_{\nu,\tau}^{(k)} = \sum_{i=0}^{k-1} X_{\nu,\tau-i}$ aggregated precipitation at k months.

By assuming precipitation at month τ with mean μ_τ, the mean of the corresponding aggregated precipitation $Y_{\nu,\tau}^{(k)}$ will be respectively:

$$\mu_\tau^{(k)} = \sum_{i=0}^{k-1}\mu_{\tau-i} \qquad (5a)$$

Also, if $\sigma_\tau^2$is the variance of precipitation at month τ, under the hypothesis of precipitation values uncorrelated in time, the standard deviation of the corresponding aggregated precipitation $Y_{\nu,\tau}^{(k)}$ will be:

$$\sigma_\tau^{(k)} = \sqrt{\sum_{i=0}^{k-1} \sigma_{\tau-i}^2(x)} \qquad \text{(5b)}$$

Substituting, Equation (4) becomes:

$$Z_{\nu,\tau}^{(k)} = \frac{\sum_{i=0}^{k-1} X_{\nu,\tau-i} - \sum_{i=0}^{k-1} \mu_{\tau-i}}{\sqrt{\sum_{i=0}^{k-1} \sigma_{\tau-i}^2}} \qquad (6)$$

Therefore, the autocovariance can be expressed as:

$$\begin{aligned} \text{cov}\left[Z_{\nu,\tau+M}^{(k)}, Z_{\nu,\tau}^{(k)}\right] &= \frac{1}{\sqrt{\sum_{i=0}^{k-1} \sigma_{\tau+M-i}^2 \sum_{j=0}^{k-1} \sigma_{\tau-j}^2}} \cdot \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} \text{cov}[X_{\nu,\tau+M-j}, X_{\nu,\tau-i}] \\ &= \frac{1}{\sqrt{\sum_{i=0}^{k-1} \sigma_{\tau+M-i}^2 \sum_{j=0}^{k-1} \sigma_{\tau-j}^2}} \cdot \sum_{i=0}^{k-M-1} \sigma_{\tau-i}^2 \qquad (7) \end{aligned}$$

By substituting Equation (7) in the variance-covariance matrix Σ, it follows:

$$\Sigma = \begin{bmatrix} 1 & \frac{\sum_{i=0}^{k-M-1} \sigma_{\tau-i}^2}{\sqrt{\sum_{i=0}^{k-1} \sigma_{\tau+M-i}^2 \sum_{j=0}^{k-1} \sigma_{\tau-j}^2}} \\ \frac{\sum_{i=0}^{k-M-1} \sigma_{\tau-i}^2}{\sqrt{\sum_{i=0}^{k-1} \sigma_{\tau+M-i}^2 \sum_{j=o}^{k-1} \sigma_{\tau-j}^2}} & 1 \end{bmatrix} \qquad (8)$$

Finally, by combining Equation (8) with Equations (1) and (2), it is possible to express the SPI transition probabilities, in terms of the variances of monthly precipitation. Although the hypothesis of normality for aggregated monthly precipitation may appear restrictive, it is worth observing that it can be justified, especially for higher values of the aggregation time scale k, as a consequence of the central limit theorem.

SPI forecasting

From a stochastic point of view, the problem of forecasting future values of a random variable is equivalent to the determination of the probability density function of future values conditioned by past observations. Once the conditional distribution is known, the forecast is usually defined as the expected value or a quantile of such distribution, and confidence intervals of the forecast values can be computed.

In practice, however, the derivation of the conditional probability distribution of future values can be cumbersome in most cases; therefore, usually a function of the past observations that forecasts future values is sought instead. More formally, let's consider a sequence of random variables $Y_1, Y_2, \ldots, Y_t$. The interest lies in determining a function $f(Y_1, Y_2, \ldots, Y_t)$ that forecast a future value Y_{t+M} with minimum error. The latter is usually expressed as the

Mean Square Error (MSE) of prediction, defined as (Brockwell and Davis, 1996):

$$\mathrm{MSE} = \mathrm{E}\left[(Y_{t+M} - f(Y_1, Y_2, \ldots, Y_t))^2\right] \qquad (9)$$

It can be shown, that the function f(•) that minimizes the MSE is the expected value of Y_{t+M} conditioned on $Y_1, Y_2, \ldots, Y_t$, i.e.:

$$f(Y_1, Y_2, \ldots, Y_t) = \mathrm{E}[Y_{t+M} \,|\, Y_1, Y_2, \ldots, Y_t] \qquad (10)$$

The above property allows one to derive the "best" forecast (in MSE sense), provided the conditional expectation can be computed. Also, it may be worthwhile to note that if Y_{t+M} is independent of $Y_1, Y_2, \ldots, Y_t$, the best predictor of Y_{t+M} is its expected value, and furthermore, the MSE of prediction is just the variance of Y_{t+M}.

Under the hypothesis of aggregated monthly precipitation normally distributed, the best predictor of the SPI M months ahead $\tilde{Z}^{(k)}_{\nu,\tau+M}$, given observations up to month τ will be:

$$\begin{aligned}\tilde{Z}^{(k)}_{\nu,\tau+M} &= E\left[Z^{(k)}_{\nu,\tau+M} \middle| Z^{(k)}_{\nu,\tau}, Z^{(k)}_{\nu,\tau-1}, \ldots\right] \\ &= E\left[\frac{\sum_{i=0}^{k-1}(X_{\nu,\tau+M-i} - \mu_{\tau+M-i})}{\sigma^{(k)}_{\tau+M}} \middle| \sum_{i=0}^{k-1} X_{\nu,\tau-i}, \sum_{i=0}^{k-1} X_{\nu,\tau-1-i}, \ldots\right] \end{aligned} \qquad (11)$$

Moreover, since conditioning on the sequence of aggregated precipitation values is equivalent to conditioning on single precipitation values, Equation(11) can be written as:

$$\tilde{Z}^{(k)}_{\nu,\tau+M} = E\left[\frac{\sum_{i=0}^{k-1}(X_{\nu,\tau+M-i} - \mu_{\tau+M-i})}{\sigma^{(k)}_{\tau+M}} | X_{\nu,\tau} = x_{\nu,\tau}, X_{\nu,\tau-1} = x_{\nu,\tau-1}, \ldots\right] \qquad (12)$$

Previous equation can be expressed as the sum of two components, namely:

$$\begin{aligned}\tilde{Z}^{(k)}_{\nu,\tau+M} = &\frac{\sum_{i=0}^{k-M-1}(x_{\nu,\tau-i} - \mu_{\tau-i})}{\sigma^{(k)}_{\tau+M}} \\ &+ E\left[\frac{\sum_{i=1}^{M}(X_{\nu,\tau+i} - \mu_{\tau+i})}{\sigma^{(k)}_{\tau+M}} | X_{\nu,\tau} = x_{\nu,\tau}, X_{\nu,\tau-1} = x_{\nu,\tau-1}, \ldots\right]\end{aligned} \qquad (13)$$

where the first term on the right hand side is referred to monthly precipitation observed in the past (i.e., from $\tau - k + M + 1$ up to the current month τ), while the second one corresponds to future values from $\tau + 1$ up to month $\tau + M$.

By assuming monthly precipitation serially independent, the above expression simplifies as:

$$\tilde{Z}^{(k)}_{\nu,\tau+M} = \frac{\sum_{i=0}^{k-M-1}(x_{\nu,\tau-i} - \mu_{\tau-i})}{\sigma^{k}_{\tau+M}} \qquad (14)$$

Note that the predictor in Equation (14) is unbiased, as can be easily verified by taking expectations on both sides.

The corresponding MSE can be computed as:

$$MSE^{(k)}_{\tau+M} = E\left[\left(\tilde{Z}^{(k)}_{\nu,\tau+M} - Z^{(k)}_{\nu,\tau+M}\right)^2\right] \qquad (15)$$

which, upon substitution of Equations (6) and (14) into (15), after some algebra becomes:

$$\begin{aligned}\mathrm{MSE}^{(k)}_{\tau+M} &= E\left[\left(\frac{\sum_{i=0}^{M-1}(X_{\nu,\tau+M-i} - \mu_{\tau+M-i})}{\sigma^{(k)}_{\tau+M}}\right)^2\right] \\ &= \left(\frac{1}{\sigma^{(k)}_{\tau+M}}\right)^2 \cdot E\left[\left(\sum_{i=0}^{M-1}(X_{\nu,\tau+M-i} - \mu_{\tau+M-i})\right)^2\right] \qquad (16)\end{aligned}$$

Since $E[\sum_{i=0}^{M-1}(X_{\nu,\tau+M-i} - \mu_{\tau+M-i})] = 0$, then:

$$\begin{aligned}E\left[\left(\sum_{i=0}^{M-1}(X_{\nu,\tau+M-i} - \mu_{\tau+M-i})\right)^2\right] &= \mathrm{var}\left[\sum_{i=0}^{M-1}(X_{\nu,\tau+M-i} - \mu_{\tau+M-i})\right] \\ &= \mathrm{var}\left[\sum_{i=0}^{M-1}(X_{\nu,\tau+M-i})\right] = \sigma^{2^{(M)}}_{\tau+M} \qquad (17)\end{aligned}$$

Thus, by making a substitution, Equation (16) simplifies as:

$$MSE^{(k)}_{\tau+M} = \left(\frac{\sigma^{(M)}_{\tau+M}}{\sigma^{(k)}_{\tau+M}}\right)^2 \qquad (18)$$

where $\sigma^{(M)}_{\tau+M}$ is the standard deviation of monthly precipitation aggregated based on the values observed at the M months preceding month $\tau + M$.

Besides MSE, a practical way of quantifying the accuracy of the forecast is by estimating the confidence interval of prediction, i.e. an interval that contains the future observed value with a fixed probability α(e.g. 95%). Obviously, the wider the interval, the less is the accuracy of the forecast and vice-versa. Confidence intervals of prediction for SPI can be estimated by capitalizing on the intrinsic normality of the index and by observing that, since the predictor is unbiased, its variance coincides with the MSE. Thus, the upper and lower confidence limits $Z_{1,2}$ of fixed probability α can be computed as:

$$Z_{1,2} = \hat{Z} \pm \sqrt{\mathrm{MSE}} \cdot u\left(\frac{1-\alpha}{2}\right) \qquad (19)$$

where, for the sake of brevity, $\hat{Z}$ represents the generic forecast and $u(\cdot)$ is the standard normal quantile corresponding to the considered probability.

Applications to precipitation series observed in Sicily

The proposed methodologies have been applied to monthly precipitation observed from 1921 until 2003, for 43 precipitation stations in Sicily. The selected stations are included in the drought monitoring bulletin published on the web-site of the Sicilian Regional Hydrographic Office (Rossi and Cancelliere, 2002, http://www.uirsicilia.it).

Evaluation of transition probabilities

By fixing several combinations of forecasting time horizon M (months) and aggregation time scales k, for each station the transition probabilities from a class of SPI at month τ to another one at month $\tau + M$ have been computed by Equation (1), for every month. In order to compute the double integral in Equation (1), which expresses the normal joint cdf (cumulative distribution function), the algorithm MULNOR has been adopted (Schervish, 1984).

The mean values of transition probabilities corresponding to the 43 stations, for $M = 6$ and $k = 24$, are presented in Figure 1, as a function of the current month τ. In particular, transition probabilities related to different combinations of initial and final drought conditions (extreme Ex, severe Se, moderate Mo, non-drought N) have been considered. In order to show also the variability transition probability among different stations, in the same plots, the limits related to ± 1 standard deviation are indicated by dashed lines. It can be observed that transition probabilities vary from one month to another, and for some transitions, also from one station to another (as indicated by the width of the limits). In particular, the mean probability value, indicated by the continuous line, of remaining in the extreme class (Ex/Ex) ranges from 60% (for February–March) to less than 25% (for August–September). The mean probability value of remaining in the non-drought class (N/N) presents a limited variability across the months, since it ranges from 95% (for February–March) to about 90% (for August–September). The transition probabilities from one class to another, for instance from extreme to non-drought or vice-versa, are very low, at least for the considered time horizon, namely $M = 6$ months. In general, it can be concluded that starting from a wet month there is an higher probability to remain in the same drought class M months ahead (plots along the diagonal), than when starting from dry months, and conversely a lower probability to return to normal conditions.

Such different behavior can be justified by considering that starting from a wet month and considering a 6 months time horizon, the occurrence probability of precipitation events, able to restore normal conditions, is very low. On the contrary, starting from a dry month, there is a high chance to observe values of precipitation such to modify drought conditions during the next 6 months.

In order to analyze the effects of the aggregation time scale k and of the forecasting time horizon M, transition probabilities computed for the 43 stations have been averaged and represented on a 3D-plot for a specific starting month and class, as a function of the final class and of the M values. In Figure 2 the case corresponding to extreme drought as starting class, August as starting month and $k = 6$, 12 and 24 is illustrated. In Figure 3 similar 3D-plots for February as starting month are shown.

It can be seen that probabilities to remain in the same class generally decrease as the forecasting time horizon M increases, while transition probabilities to non-drought condition show an opposite behaviour. Further, as the aggregation time scale k increases, the probabilities of remaining in the same class increase, whereas transition probabilities to return to non-drought condition generally decrease. Finally, by comparing transition probabilities

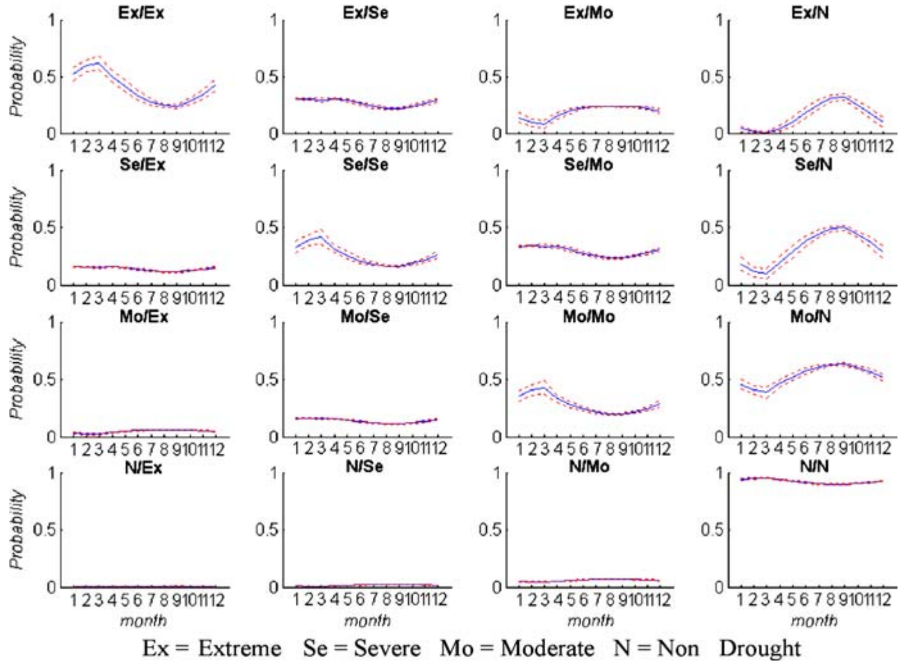

Fig. 1 Mean of transition probabilities (continuous line) for $M = 6$ and $k = 24$ computed on 43 stations, and limits corresponding to ± 1 standard deviation (dashed line)

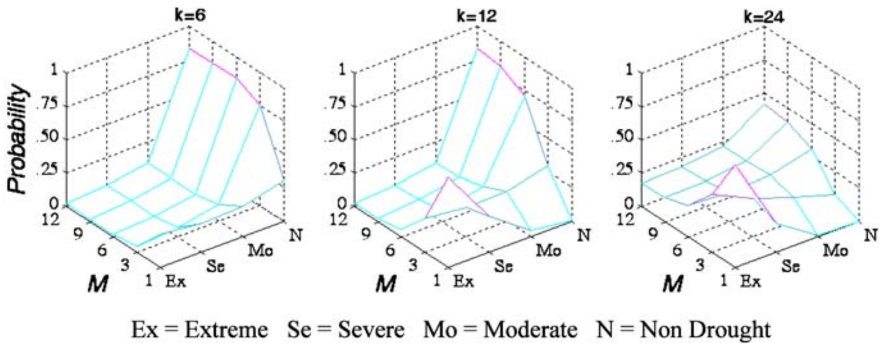

Fig. 2 Mean transition probabilities (computed on the 43 stations) as a function of forecasting time horizon M (starting class: extreme drought (Ex), starting month: August)

related to August to those corresponding to February, it can be inferred that it is generally easier to recover from drought conditions starting from August than starting from February.

Finally, an analysis of the spatial variability of transition probabilities in Sicily, has been also carried out for the 43 stations, by considering an aggregation time scale $k = 12$ months and $M = 3$, 6 and 9 months, with reference to a typical wet month, February, and a dry month, August. In particular, by interpolating the values of transition probabilities derived for each station by means of the IDW method (Inverse Distance Weighted), maps of the spatial distribution of transition probabilities over Sicily region have been obtained.

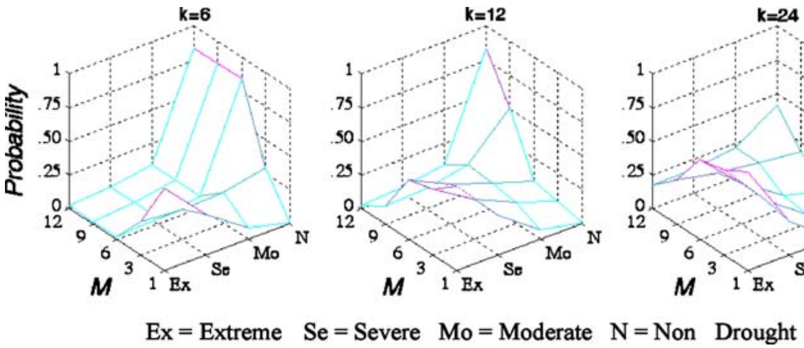

Fig. 3 Mean transition probabilities (computed on the 43 stations) as a function of forecasting time horizon *M* (starting class: extreme drought (Ex), starting month: February)

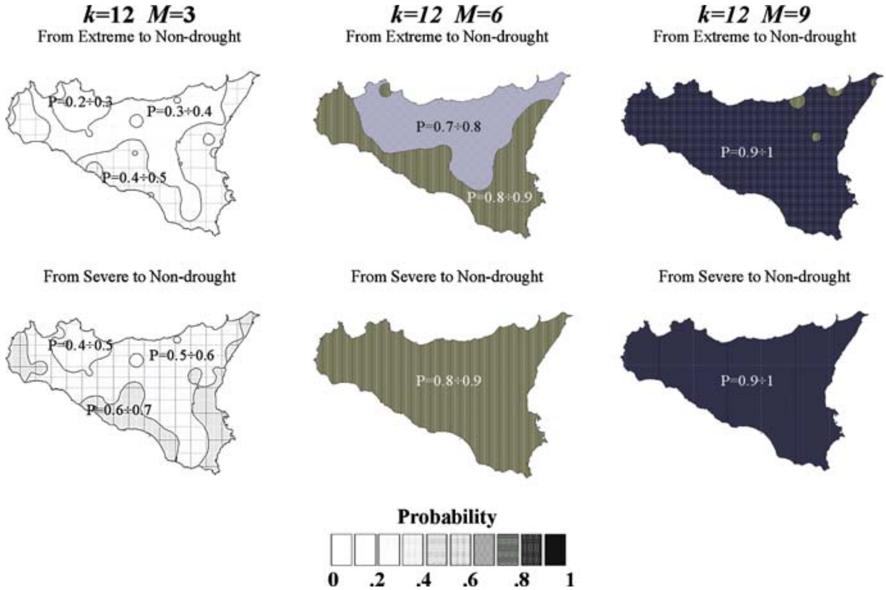

Fig. 4 Spatial distribution of transition probabilities from a drought class to a non-drought class (Starting month: August)

In Figures 4 and 5, maps related respectively to August and February, as starting months, are shown. It can be concluded that the probability to return to a normal condition increases as the forecasting time horizon *M* increases and the starting drought condition decreases. For the case of August, it can be observed that the north-central part of Sicily is generally characterized by lower values of probabilities for returning to normal condition with respect to the rest of the island, while, for the case of February, the same area is characterized by higher values than anywhere else. Such an opposite behaviour can be partially explained by the different pluviometric regime in the different areas, which affects the covariance term in Equation (7) and in turn, transition probability values.

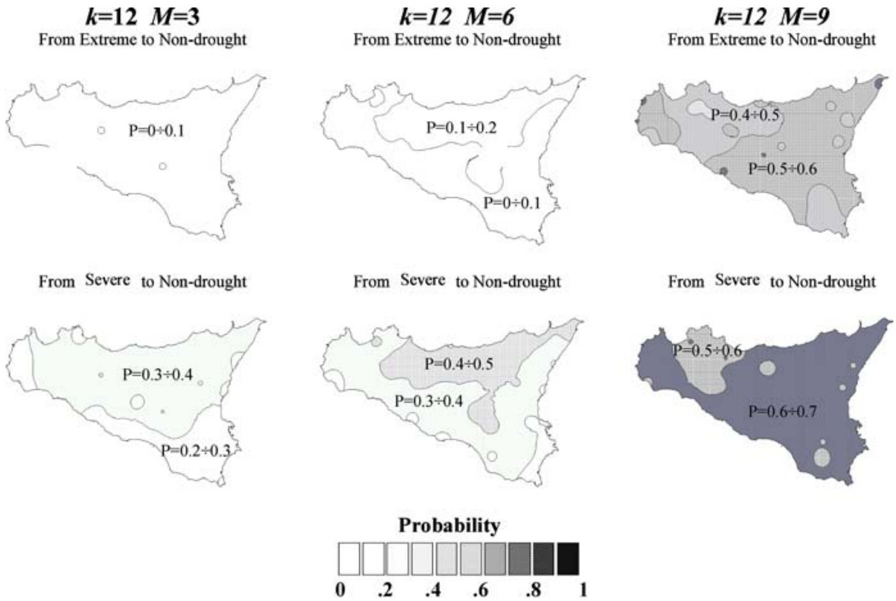

Fig. 5 Spatial distribution of transition probabilities from a drought class to a non-drought class (Starting month: February)

Need of analytical approach for deriving transition probabilities

Despite the apparent complexity of the proposed analytical approach for estimation of transition probabilities, it should be pointed out that the methodology yields results that can be considered in general more reliable than those obtained by alternative approaches, such as frequency analysis of observed transitions in an historical sample, or application of a Markov chain scheme. With regard to the frequency approach, it should be underlined that the limited sample size of observed transitions among different SPI classes, even in the case of rather long records of monthly precipitation, hinders the possibility of reliable frequency estimates.

As an example, Table 2 reports the number of transitions, observed from February to August, among SPI classes computed on precipitation series observed for Caltanissetta aggregated at $k = 3$, 6, 9, 12 and 24 months (80 years). It can be inferred that the number of observed transitions is generally not sufficient to compute reliable frequency estimates. Furthermore, the lack of observed transitions in some cases (e.g. going from Extreme conditions in February to any other conditions in August) would lead to the misleading conclusion that such transitions have zero occurrence probability, which is obviously not correct. Application of the analytical approach on the other hand, allows one to always estimate transition probabilities, even from relatively short records, since the whole available precipitation series, and not just the few observed transitions, are utilized.

Regarding the applicability of Markov chain hypothesis to model transitions of SPI values from one drought class to another, it should be mentioned that such an assumption may not be valid in general. Indeed, under the non-homogeneous lag-1 Markov hypothesis, the lag-M transition probability matrix $\Pi(M)_\tau$, whose generic element (i, j) is given by

Table 2 Number of transitions between drought/non-drought classes starting in February, ending in August, computed from SPI series ($k = 3, 6, 9, 12, 24$ months) of Caltagirone station, Sicily (80 years)

		Drought class in August				
	Drought class in February	Extreme	Severe	Moderate	Non drought	Total
$k = 3$	Extreme	0	0	0	3	3
	Severe	0	0	0	2	2
	Moderate	0	0	0	4	4
	Non drought	0	4	10	57	71
$k = 6$	Extreme	0	0	0	1	1
	Severe	0	0	0	3	3
	Moderate	1	0	1	6	8
	Non drought	1	0	10	57	68
$k = 9$	Extreme	0	0	0	0	0
	Severe	0	1	1	1	3
	Moderate	1	0	3	6	10
	Non drought	0	0	9	58	67
$k = 12$	Extreme	0	0	1	0	1
	Severe	0	1	0	3	4
	Moderate	0	2	0	5	7
	Non drought	0	3	5	60	68
$k = 24$	Extreme	0	0	0	0	0
	Severe	0	5	3	2	10
	Moderate	1	0	2	0	3
	Non drought	0	0	3	64	67

$P[Z^{(k)}_{\nu,\tau+M} \in C_j | Z^{(k)}_{\nu,\tau} \in C_i]$, can be written, for fixed k, as (Bremaud, 1999):

$$\Pi(M)_\tau = \Pi_\tau \Pi_{\tau+1} \ldots \Pi_{\tau+M-1} \qquad (20)$$

where Π_τ is the lag 1 transition probability matrix whose generic element (i, j) is given by $P[Z^{(k)}_{\nu,\tau+1} \in C_j | Z^{(k)}_{\nu,\tau} \in C_i]$.

Note that the non homogenous formulation is required for the SPI, since the homogenous one would obviously not be able to model the general strong seasonal pattern observed in transition probabilities (see Figure 1).

In order to verify whether the SPI can be modeled by a Markov chain, the (exact) lag-M transition probability matrix $\Pi(M)_\tau$ computed by means of Equation (1), has been compared with the one obtained, under the Markov hypothesis, by Equation (20). As an example, in Tables 3 and 4, the percentage differences between each element of the two matrices are shown with reference to Caltagirone station and to the transitions from February and August with time horizon $M = 3$, 6 months, and at $k = 9$, 12, 24 months. From the table, it can be inferred that the differences between the two approaches are generally not negligible. Furthermore, since the analytical approach can be considered exact, such differences can be interpreted as percentage errors in computing transition probabilities when a Markov chain model is adopted. For instance, under the Markov hypothesis, the percentage error in the probabilities of transition from Extreme condition in February to Extreme condition in May are generally in the order of 15%. In other cases, the errors appear

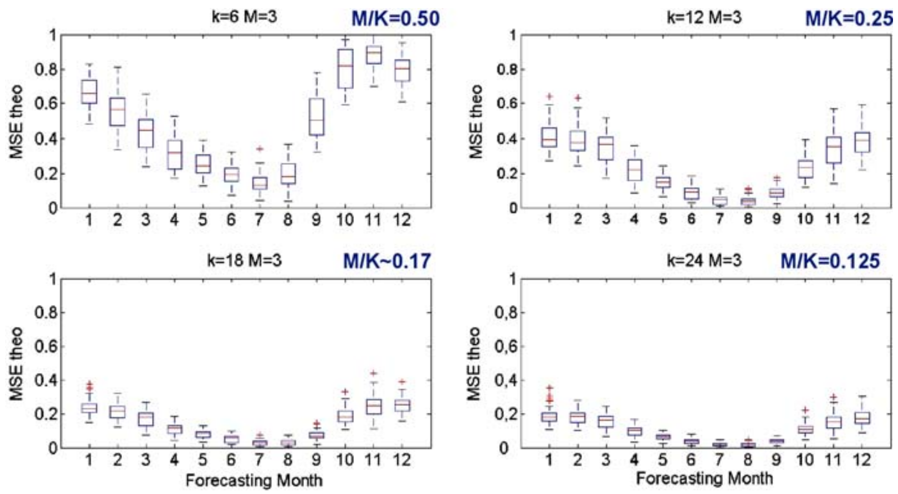

Fig. 6 Theoretical values of SPI's MSE for different k and M and different ratios M/k, under the hypothesis of normally distributed aggregated precipitation series

much larger, although the errors related to the Normal-Normal transition are always very low.

Similar results have also been obtained by considering other values of M and k and different stations.

The above errors, coupled with the unreliability of the frequency estimation of transition probabilities, clearly indicates the practical value of the proposed analytical approach, which can find application even when relatively short records of precipitation are available.

SPI forecasts

The forecasting model proposed has been applied by considering several aggregation time scales, $k = 6, 9, 12$ and 24 months, as well as different forecasting time horizons $M = 3, 6, 9$ and 12 months.

First, theoretical MSE values (see Equation (18)) have been computed for all 43 stations. Figure 6 illustrates the boxplots of monthly MSE values for different aggregation time scales k and time horizon $M = 3$ months. The overall height of each boxplot indicates the variability of MSE among the different stations. As expected, the performance of the forecasting model gets better (lower MSE's) as the ratio of M over k decreases. The effect of seasonality is evident in the considered cases, although with different patterns according to the scale of aggregation k. Clearly such an effect disappears if k and M are multiples or equal to 12 months, so that MSE's remain more or less the same, regardless of the forecasting month (see Figure 7).

The forecasting model has been validated by comparing observed and forecasted SPI (computed by Equation 14) during a period different from that used for parameters estimation. Such validation is usually carried out by splitting the available sample into two sub-samples to be used for parameter estimation and model validation respectively (Klemes, 1986). Here a slightly different approach is proposed, where the generic SPI value at a given time interval is compared with the corresponding forecast, computed by estimating the parameters on the previous N years. Thus, a period of past N years is considered for parameters estimation every year. This is consistent with the fact that when the model is applied for real time forecast, its

Table 3 Percentage differences of transition probability matrices computed exactly by Equation (1) and under the lag-1 Markov hypothesis by Equation (20) for different k and M, for Caltagirone station. Starting month: February

		$k=9$				$k=12$				$k=24$			
		Ex	Se	Mo	N	Ex	Se	Mo	N	Ex	Se	Mo	N
$M=3$	Ex	15.8	−8.0	−22.7	−78.7	15.1	−6.5	−9.9	−42.1	15.4	−9.6	−50.7	−206.7
	Se	−7.5	12.7	10.4	−35.6	−5.7	9.6	12.7	−22.7	−9.5	15.3	6.5	−64.2
	Mo	−24.1	9.8	26.4	−21.4	−11.6	11.7	26.5	−18.2	−51.3	6.3	24.3	−24.5
	N	−77.9	−34.0	−21.7	1.6	−41.4	−21.0	−18.7	1.5	−206.0	−63.5	−24.6	1.6
$M=6$	Ex	2.2	−9.0	3.8	1.2	26.4	0.3	−3.6	−70.6	27.9	−5.2	−52.6	−309.5
	Se	−11.3	−1.0	15.2	−3.6	1.7	16.3	25.4	−34.6	−4.7	24.1	19.1	−90.1
	Mo	8.2	18.9	27.6	−11.2	−6.9	23.4	38.1	−23.4	−54.0	18.6	38.7	−34.9
	N	−0.3	−5.5	−10.3	1.0	−68.4	−32.2	−24.3	2.4	−307.8	−88.6	−35.2	2.8

Table 4 Percentage differences of transition probability matrices computed exactly by Equation (1) and under the lag-1 Markov hypothesis by Equation (20) for different k and M, for Caltagirone station. Starting month: August

		$k = 9$				$k = 12$				$k = 24$			
		Ex	Se	Mo	N	Ex	Se	Mo	N	Ex	Se	Mo	N
$M = 3$	Ex	−8.8	−10.1	3.1	3.7	8.0	−4.8	1.5	−4.9	15.0	−5.6	−11.5	−42.0
	Se	−11.1	−3.1	9.6	−0.8	−7.1	1.6	12.2	−5.2	−6.4	9.6	11.6	−21.0
	Mo	4.6	11.1	17.8	−5.5	4.6	15.1	24.3	−11.5	−9.9	12.5	26.2	−18.5
	N	3.4	−1.4	−5.2	0.5	−5.5	−7.3	−10.7	1.0	−42.7	−22.6	−18.1	1.5
$M = 6$	Ex	−86.6	−51.8	−14.2	9.9	−40.5	−29.2	−0.2	8.9	20.7	1.0	8.1	−27.0
	Se	−51.4	−28.9	−6.0	4.8	−30.2	−15.7	5.8	2.8	0.9	10.4	24.7	−17.0
	Mo	−17.2	−7.9	1.4	0.9	0.2	6.5	14.3	−3.0	8.1	24.7	35.4	−17.3
	N	10.3	5.1	0.7	−0.6	9.0	2.6	−2.9	0.0	−26.9	−17.0	−17.3	2.0

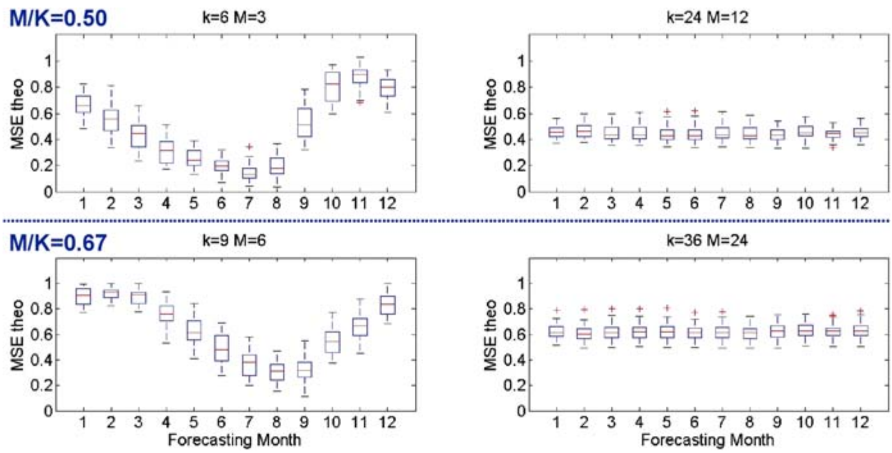

Fig. 7 Effect of seasonality in theoretical values of SPI's MSE for different k and M and equal ratios M/k, under the hypothesis of normally distributed aggregated precipitation series

Table 5 Performance of the forecasting model for $k = 6$, 12 and 24 months and $M = 3$ months (Caltagirone station)

Performance indices	$k = 6$ and $M = 3$	$k = 12$ and $M = 3$	$k = 24$ and $M = 3$
r	0.715	0.850	0.895
RMSE	0.731	0.565	0.466
MAD	0.551	0.436	0.363

parameters are usually computed on the basis of the last two or three decades of observation. According to the results of a previous sensitivity analysis (Cancelliere *et al.*, 2005), a moving window of 20 years has been selected for model validation.

As an example, Figure 8 shows the comparison between observed and forecasted values SPI for one of the 43 stations, namely Caltagirone, for different combinations of the time scales k and $M = 3$ months. On the same plots, 95% confidence intervals, estimated by means of Equation (19) are also shown.

As expected, the accuracy of the forecast decreases as larger k are adopted with respect to the fixed M. Nonetheless, from the figure it can be inferred a fairly good agreement between observed and forecasted SPI values for all the considered cases, as is also evident from the fact that almost all of the observed values lie within the confidence intervals. In particular, such agreement is confirmed by the values of the indices used to evaluate the performance of the forecasting models (see Table 5), namely:

- r = correlation coefficient between the observed $Z^{(k)}_{\nu,\tau}$ and the forecasted $\hat{Z}^{(k)}_{\nu,\tau}$ series;
- RMSE = root mean square of errors of forecasting $\sqrt{\sum_{i=1}^{N} (Z^{(k)}_{\nu,\tau} - \hat{Z}^{(k)}_{\nu,\tau})^2/n}$ where n is the number of forecasted values;
- MAD = mean absolute deviation between the observed and the forecasted values $\sum_{i=1}^{N} |Z^{(k)}_{\nu,\tau} - \hat{Z}^{(k)}_{\nu,\tau}|/N$

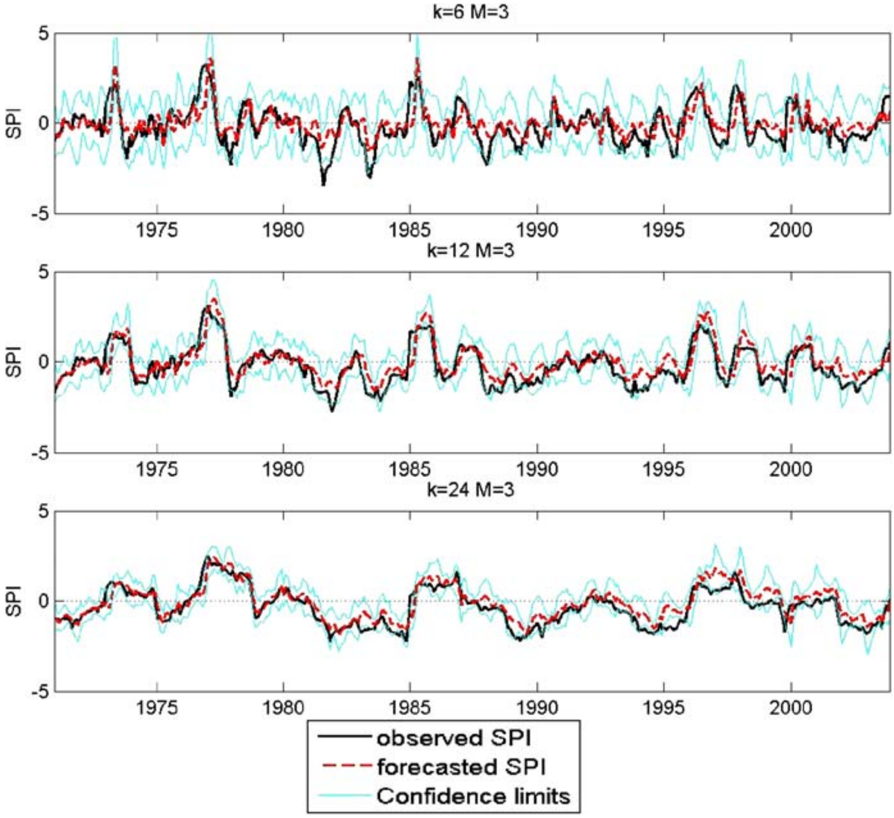

Fig. 8 Model validation: comparison between observed and forecasted SPI for Caltagirone station (moving window: 20 years)

Conclusions

Drought monitoring and forecasting are essential tools for implementing appropriate mitigation measures in order to reduce negative impacts. Knowledge of transition probabilities from a drought class to another, for a given site or region, as well as the availability of forecasts of drought indices, and of the related confidence intervals, can help to improve the decision making process for drought mitigation, since appropriate measures can be selected based on the risk associated with the possible evolution of a current drought condition.

In the paper, stochastic methodologies (i) to compute drought transition probabilities, based on the SPI index, and (ii) to forecast future SPI values on the basis of past precipitation, are presented. Analytical expressions of the Mean Square Error of prediction are also presented, which enables one to derive confidence intervals for the forecasted values.

The effects of the aggregation time scale k and of the forecasting time horizon M on transition probabilities have been analyzed for 43 stations, by fixing the starting month and class, as a function of the final class and of the M values. Also, an analysis of the spatial distribution of transition probabilities in Sicily, shows distinguished areas characterized by a different tendency to return to normal conditions after experiencing drought.

The proposed methods to compute transition probabilities is particularly valuable from a practical standpoint, in light of the difficulties of applying a frequency approach due to the limited number of transitions generally observed even on relatively long SPI records. Also, an analysis of the applicability of a Markov chain model has revealed the inadequacy of such an approach, since it leads to significant errors in the transition probability.

Validation of the forecasting model has been carried out by comparing SPI values computed on precipitation observed in the same 43 stations in Sicily and the corresponding forecasts, making use of a moving window scheme to estimate the parameters of the model. The results show a fairly good agreement between observations and forecasts, as it has also been confirmed by the values of some performance indices, which indicates the suitability of the model for short- medium term forecast of drought conditions.

Ongoing research is being carried out to improve the forecasting capabilities of the model, by taking into account exogenous covariates such as large scale climatic indices.

Acknowledgements The present research has been developed with the partial financial support of the EU Projects "Mediterranean drought preparedness and mitigation planning" (MEDROPLAN, EuropeAid MEDA, contract no. ME8/AIDCO/2001/0515/59770-P027) and of the Italian project MIUR PRIN 2005 "Previsione e mitigazione della siccità".

References

Bonaccorso B, Bordi I, Cancelliere A, Rossi G, Sutera A (2003) Spatial variability of drought: an analysis of SPI in Sicily. Water Resour Manag 17:273–296

Bordi I, Fraedrich K, Petitta M, Sutera A (2005) Methods for predicting drought occurrences. In: Proc. of the 6th international conference of the European water resources association Menton, France, pp 7–10 September 2005

Bremaud P (1999) Markov chains, Gibbs fields, Montecarlo simulation and queues, Springer, The Netherlands

Brockwell PJ, Davis RA (1996) Introduction to time series and forecasting Springer-Verlag ed., New York, pp 1–422

Cancelliere A, Di Mauro G, Bonaccorso B, Rossi G (2005) Stochastic forecasting of standardized precipitation index. Proc. of XXXI IAHR Congress "Water engineering for the future: choice and challenges". Seoul, Korea, 11–16 September 2005, pp 3252–3260

Cancelliere A, Rossi G, Ancarani A (1996) Use of Palmer Index as drought indicator in Mediterranean regions. Proc. IAHR Congress "From flood to drought". Sun City, South Africa, August 5–7, 1996, S4.12–1–25

Guttman NB, (1998) Comparing the Palmer drought severity index and the standardized precipitation Index. J Amer Water Res Ass 34(1):113–121

Guttman NB (1999) Accepting the standardized precipitation index: a calculating algorithm. J Amer Water Res Ass 35(2):311–323

Hayes MJ, Svoboda M, Wilhite DA, Vanyarkho O (1999) Monitoring the 1996 drought using the SPI. Bull Amer Meteor Soc 80:429–438

Heim RR (2000) Drought indices: a review. In: DA Wilhite (ed) Drought: a global assessment. Routledge, London and New York

Karl T, Quinlan F, Ezell DS (1986) Drought termination and amelioration: its climatological probability. J Climate Appl Meteor 26:1198–1209

Keyantash J, Dracup JA (2002) The quantification of drought: an evaluation of drought indices. Bull Amer Meteor Soc 83(8):1167–1180

Klemes V (1986) Operational testing of hydrological simulation models. Hydrol Sci J 31(1):13–24

Lohani VK, Loganathan GV, Mostaghimi S (1998) Long-term analysis and short-term forecasting of dry spells by Palmer drought severity index. Nord Hydrol 29(1):21–40

McKee TB, Doesken NJ, Kleist J (1993) The relationship of drought frequency and duration to time scales. In: Proc. 8th conference on applied climatology. Anaheim, California, pp 179–184

Mood AM, Graybill FA, Boes E (1974) Introduction to the theory of statistics. McGraw-Hill, New York

Rossi G, Cancelliere A (2002) Early warning of drought: development of a drought bullettin for Sicily. Proc. 2nd international conference "New trends in water and environmental engineering for safety and life: eco-compatible solutions for aquatic environments". Capri, Italy, June 24–28, 2002, pp 1–12

Rossi G (2003) Requisites for a drought watch system. In: G. Rossi *et al*. (ed), tools for drought mitigation in Mediterranean regions. Kluwer Academic Publishing, Dordrecht, pp 147–157

Schervish MJ (1984) Algorithm AS 195. Multivariate normal probabilities with error bound. Applied Statistics 33(1):81–94

Wilhite DA, Hayes MJ, Svoboda MD (2000) Drought monitoring and assessment: status and trends in the United States. In: Vogt JV, Somma F (eds) Drought and drought mitigation in Europe. Kluwer Academic Publishers, pp 149–160